Another Look at the Optimum Frequencies for a Water Vapor Radiometer

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A water vapor radiometer is used to estimate the columnar content of atmospheric water vapor or equivalently the line-of-sight path delay due to water vapor. Two measurement channels are used in order to separate the effects of the liquid and vapor phases of water. The efficiency of the path delay or columnar vapor estimate is dependent on the choice of on-line frequency channel. Previous analysis of this problem has suggested frequencies from 20.3 to 21 GHz. The frequency that yields the minimum error in the inversion algorithm is shown here to be both site and season dependent. Hence, the concept of an "optimum" frequency must represent an averaging process over the entire range of meteorological conditions that is expected. For a range of sites and conditions representing a cross section of the continental United States the optimum on-line frequency seems to be 20.6 GHz.

I. Introduction

Interferometic techniques at microwave frequencies are being used in the fields of geodesy and spacecraft navigation (Ref. 1). The time delay imposed by varying amounts of atmospheric water constitutes a potentially limiting error source for these techniques. This error can be reduced by estimating the line-of-sight delay due to water vapor using a device known as a water vapor radiometer (WVR).

Water vapor in the Earth's atmosphere slows the passage of an electromagnetic wave. Equivalently, the electrical path through the medium is longer than the vacuum path (Ref. 2) by the amount ΔL_v, where,

$$\Delta L_v \text{ (cm)} = 1.723 \times 10^{-3} \int \frac{\rho_v}{T} \, ds \qquad (1)$$

where ρ_v is measured in g/m^3, s in meters, and a line of sight through the entire atmosphere is assumed. Water vapor in the Earth's atmosphere emits spectral line radiation centered at a frequency of 22.235 GHz. If the opacity of the atmosphere is low, then to a good approximation the brightness temperature measured on the vapor line is proportional to the columnar vapor content along the line of sight. Hence, if atmospheric opacity or brightness temperature can be measured, Eq. (1) can be estimated. These measurements can be accomplished with passive microwave sensing techniques (Ref. 3) using a device known as a water vapor radiometer (WVR). Typically, the WVR includes a second measurement channel somewhat offset from the water vapor line that is used to eliminate the effects of liquid water in the form of clouds.

The water vapor line is pressure broadened so that the detailed line shape is a function of the vertical distribution of vapor. Since water vapor is *not* a well-mixed constituent of the atmosphere, it will vary according to site, season, and local meteorological conditions. Both the point value of the vapor density and its distribution vertically and horizontally will vary on a variety of time and spatial scales. The full width of the vapor line at half intensity is approximately 4 GHz whereas the typical observing bandwidth of a WVR is 0.1 - 0.5 GHz. This implies that the accuracy with which Eq. (1) can be estimated will be somewhat dependent upon the exact frequency choice of the WVR. Previous workers have suggested the "optimum" frequency for the WVR is between 20.3 and 21.0 GHz. In this paper the concept of an "optimum" frequency choice is examined along with its site and seasonal variations using a meteorological data base representing a cross section of conditions of the continental United States.

II. The Problem

Section I established the problem as the estimation of the integral quantity,

$$\Delta L_v = 1.723 \times 10^{-3} \int \frac{\rho_v}{T} \, ds$$

The spectrum of atmospheric emission around the 22 GHz water vapor line is shown in Fig. 1 for two different vertical distributions of vapor. The pressure distribution in a standard atmosphere was used with a surface temperature of 30°C and the standard lapse rate. In order to calculate Fig. 1(a), a constant relative humidity between 0 and 1 km is assumed. For Fig. 1(b), a constant relative humidity between 2 to 3.8 km altitude is assumed. The columnar water vapor content was very nearly the same in both cases and equal to 2 g/cm^2. The high altitude vapor clearly shows a sharper line than the low altitude profile although the path delay is the same in both cases. This presents a tradeoff decision in regard to the operating parameters of a WVR. If we wish to maximize the signal from a given amount of vapor, then clearly we should observe at the line center, i.e., f = 22.225 GHz. Equally clear is the fact that we would also be most sensitive to variations in the vapor profile at this frequency. Signal detection is not in general a problem if standard radiometric techniques are used so we will choose to minimize the effects of variations in the vapor profile. Inspection of Fig. 1 would suggest that a single frequency measurement of the brightness temperature somewhere near the half-power point of the line profile would provide the most accurate estimate of the path delay.

Figure 2 shows the brightness temperature of the atmosphere between 10 and 40 GHz plotted for three different cases. Figure 2(a) shows the spectrum of a standard atmosphere without water vapor or liquid. In this case the emission is primarily from the wings of a complex of oxygen emission lines near 60 GHz and is called emission from the "dry" atmosphere. In Fig. 2(b) an exponential distribution of vapor has been added having a columnar content M_v = 2 g/cm^2, and the spectral line centered at 22.2 GHz is clearly evident. Figure 2(c) shows the same atmosphere as in 2(b) but with an additional M_L = 0.1 g/cm of liquid water that is assumed to exist in the form of small droplets.

Figure 2(c) shows that a relatively small amount of liquid water (i.e., such as exists in clouds) can cause a large change in the observed brightness temperature. Crane (Ref. 4) has estimated the effective dielectric constant for liquid water in the form of clouds or light precipitation to be 1.6 so that the excess path delay due to liquid water is approximately

$$\Delta L_L = 1.6 \, M_L \tag{2}$$

where M_L is the total precipitable liquid water along the line of sight in g/cm^2. Since the liquid water from most clouds translates to a precipitable liquid column that is a few tens or hundreds of microns, the path delay due to liquid water is negligible. If there is enough liquid in the atmosphere to contribute to the path delay, then the inversion algorithms probably are not applicable.

Although the liquid water found in clouds does not contribute to the path delay, it does contribute to the brightness temperature – the main observable of the WVR. Droplets of liquid water act as scattering centers removing energy from the line-of-sight path and injecting scattered power from other sources, i.e., the Earth at T = 290 K. If the water droplets are small compared to the wavelength of the radiation and the integrated liquid content is not too large, then the assumptions regarding a non-scattering, low opacity medium will not be violated. This means that the path delay can be estimated if the brightness temperature effects of the liquid and vapor state of water can be separated. A second channel is necessary in the WVR solely to separate the liquid water and vapor effects. The frequency of this second channel should be such as to provide good sensitivity to liquid and maximum contrast to the vapor measurement. Inspection of Fig. 2 suggests that a frequency between 30-32 GHz in the atmospheric "window" would be the appropriate choice for the off-line channel. The frequency of 31.4 was chosen because (a) it is on the wing of the water vapor line so that it is relatively insensitive to vapor but is sensitive to liquid and (b) that frequency band is allocated to space research.

Resch (Ref. 5) has shown that a general formulation of the inversion algorithm for the excess path delay due to water vapor ΔL_v, can be expressed as

$$\Delta L_v = \left[G_1 - \left(\frac{f_1}{f_2}\right)^2 G_2\right]^{-1}$$

$$\times \left[\tau_1 - \left(\frac{f_1}{f_2}\right)^2 \tau_2 - (1-\beta)\,\tau_{d1}\right] \tag{3}$$

where τ_i is the observed opacity at frequency f_i, and τ_d is the opacity due to the "dry" atmosphere, i.e., primarily oxygen, and β is the ratio of τ_d at the two observing frequencies. The observable quantity with a WVR is the brightness temperature T_{bi}, which is transformed to an opacity using the relation

$$\tau_i = -\log_e \left(\frac{T_{mi} - T_{bi}}{T_{mi} - T_c}\right) \tag{4}$$

where T_{mi} is the mean radiating temperature of the atmosphere, and T_c is the cosmic blackbody background (2.9 K). The quantity G_i is termed the single-frequency weighting function and is defined as

$$G(f, s) = (5.803 \times 10^2) \left(\frac{T\alpha_v}{\rho_v}\right) \tag{5}$$

where T is the physical temperature (Kelvin), ρ_v is the vapor density (g/m^2), and α_v is the vapor absorption coefficient (neper/m). In the inversion algorithm we have the quantity W defined as,

$$W = \left[G_2 - \left(\frac{f_2}{f_1}\right)^2 G_1\right] \tag{6}$$

and called the dual frequency weighting function, or simply the weighting function.

Previous workers have attacked the problem of optimum frequency choice by examining the behavior in either G_i or W and picking the frequency of minimum variation. Menius et al. (Ref. 6) considered the single frequency weighting function in a standard atmosphere with an assumed vapor distribution that represented "average" conditions and concluded that the operating frequency should be either 20.8 or 23.7 GHz. Westwater (Ref. 7) also examined the single frequency weighting function but used two radiosonde launches chosen at random from Miami, Florida. He concluded that G_i is relatively flat for the entire frequency interval 20.1 to 21.0 GHz and that the slope closest to zero lies between 20.6 and 20.8 GHz. Gaut (Ref. 8) in his Ph.D. thesis did not directly address the optimum frequency choice but did give an excellent discussion of the problem using data representative of yearly variations in the New England area and pointed out that with three frequencies one can have a composite weighting function that is constant to a high degree of approximation. Webster (Ref. 9) employed a simulation analysis to determine the optimum frequencies of a two channel WVR using model atmospheres to solve the equation of radiative transfer. He argued that regardless of the off-line frequency, the on-line measurement should be about 1 GHz from the line center, i.e., 21 or 23 GHz. Wu (Ref. 10) systematically searched for minimum variation in W for one radiosonde launch from Pt. Mugu, California, and concluded that the frequency pair 20.3 and 31.4 GHz was optimum. He examined data from two additional launches, representing dry and wet conditions and argued that his conclusion was consistent with these data.

One might wonder whether these different conclusions are due to different analysis, different input data, or both. The next section will show that "flattest" values for both G_i and W are site and season dependent. Hence the "optimum" frequency choice for the on-line measurement channel must be made after consideration of the entire range of meteorological conditions under which our instrumentation is expected to perform.

III. Analysis of the Problem

The quantity of fundamental importance in our calculations is the total absorption coefficient evaluated at some arbitrary point s along a given line of sight through the atmosphere. The analytical formulation for the absorption coefficient used in the following discussion is taken from Ref. 11.

In order to evaluate the integral quantities we must have the distribution functions $T = T(s)$, $P = P(s)$, $\rho_v = \rho_v(s)$, etc. Depending upon immediate objectives and/or convenience, these distributions can be generated in either of two ways. The first method is to use the "standard atmosphere" (Ref. 12) to which we add an assumed exponential distribution of water vapor specified by a surface density ρ_o, and scale height h_o, of the form

$$\rho_v = \rho_o \exp\left(-\frac{h}{h_o}\right) \tag{7}$$

so that the total precipitable vapor content is simply

$$M_v = \rho_o h_o \tag{8}$$

The second method is to use radiosonde data from several U.S. sites to generate a table of T, P, and ρ_v sampled at standard levels.

Using the standard atmosphere we can investigate how various quantities vary under nominal conditions. Using the radiosonde data we can investigate how these quantities vary in the real atmosphere in regard to site and season. The data base of radiosonde data is taken from five sites in the United States during the year 1976. The sites – Portland, Maine; Pittsburgh, Pennsylvania; El Paso, Texas; San Diego, California; and Oakland, California – were chosen to represent a cross section of meteorological conditions. Each radiosonde launch provides an approximately vertical profile of pressure, temperature, and relative humidity that is used to calculate the weighting functions. One launch out of eight was selected from each site so as to obtain equal amounts of data from both 0 and 12 hours Universal Time and to cover seasonal trends in the data.

Figure 3 shows the variation of the single-frequency weighting function with altitude for several frequencies in the vicinity of the water-vapor line. It is obvious that $G(f = 31.4$ GHz$)$ is not constant, nor for that matter is G_i for any other frequency. This observation raises several questions in light of our previous assumptions. First, how badly have we violated our assumption regarding the consistency of G_i? Second, if G_i is not a constant with regard to s, how do we pick the "best" frequency to measure the on-line brightness temperature? Third, since we know that G_i is not really a constant, what value shall we take to represent W in Eq. (3); the surface value, the value at 1 km, an average value?

Consideration of the behavior of the single-frequency weighting function provides insight in regard to systematic error sources. For a single channel WVR we would choose an operating frequency on the vapor line that yielded the "flattest" G_i for all of the sites that we intend to visit. For a two channel WVR the situation is somewhat more complicated in that we desire the best inversion accuracy; hence, we must minimize the variance of W, not G_i. If we assume that the off-line measurement is made at a frequency of 31.4 GHz, then the "best" frequency for the vapor channel is found by computing $\Delta W/\overline{W}$ versus frequency for radiosonde data and noting the frequency at which this quantity is a minimum. Figure 4 shows $\Delta W/\overline{W}$ versus frequency, calculated for a standard atmosphere containing an exponential distribution of vapor with a scale height of 2.2 km, where $\overline{W}$ is the average value of W and ΔW is the rms value. The minimum value of $\Delta W/\overline{W}$ will depend on the altitude at which we truncate the exponential distribution of the water vapor. The three curves in Fig. 4 illustrate what happens to $\Delta W/\overline{W}$ as the vapor distribution is tuncated at altitudes of 11 km, 6 km, and 4 km. Note that the frequency of the minimum variation point is a function of the cutoff altitude. Since the vapor distribution in reality tends to be both site and season dependent, it suggests that the concept of an "optimum" frequency for the vapor channel is likely to be site and season dependent. This is probably the reason why previous workers have suggested slightly different values for the optimum frequencies of a WVR. Also note in Fig. 4 that the percentage deviations represented by $\Delta W/\overline{W}$ are not very large – even at the 22.235 GHz frequency. Hence, the assumption regarding the constancy of G_i is reasonably good.

Radiosonde data shows that the altitude distribution of water vapor changes from day to day. Figure 5(a) shows f_m equals the frequency of minimum $\Delta W/\overline{W}$ vs. date for the Portland, Maine, radiosonde data. Figure 5(b) shows the same quantity calculated from the El Paso, Texas, radiosonde data. For these figures, $\overline{W}$ is calculated only at those points where the radiosonde indicates the presence of water vapor. The figures illustrate that the optimum frequency of the WVR is indeed site and season dependent. In our geodetic support applications the WVRs are expected to operate at a wide variety of sites throughout the year. The best frequency choice can only be optimum in some average sense. Figures 5(c) through 5(h) show the distribution function for f_m calculated from all five radiosonde launch sites and suggest that the best choice for the vapor frequency under continental U.S. conditions should be 20.6 GHz.

For historical reasons (Ref. 13), the operating frequencies of the WVRs recently constructed at JPL in order to support VLBI experiments are 20.7 and 31.4 GHz. This means that the error in the path delay estimates from these instruments may depend on the vapor profile a bit more strongly than if we had chosen the lower operating frequency. Figure 6(a) shows the distribution of $\overline{W}$ calculated from radiosonde data for Portland, and Fig. 6(b) shows the histogram for all sites. The time plots of this data for all sites show a clear seasonal dependence that raises the possibility that the variations in W can be reduced by modeling. Inspection of Eqs. (3) and (5) suggests that it might be possible to empirically parameterize W in the form $W = W(P, T, \rho_v)$, i.e., as a function of surface parameters, so as to further reduce these variations.

IV. Summary

The formulation of an inversion algorithm for use with a water vapor radiometer (Eq. 3) contains a term of the form,

$$W = \left[G_1 - \left(\frac{f_1}{f_2} \right)^2 G_2 \right]$$

that we call the dual frequency weighting function. The terms G_1 and G_2 are the single frequency weighting functions associated with two measurement channels of the WVR and were assumed to be constant in the derivation of the algorithm. We have seen that quantities are not truly constant in the real atmosphere and hence introduce "noise" in the inversion process. If one wishes to minimize the level of this noise and (1) accepts the fact that the off-line channel is f_2 = 31.4 GHz, and (2) uses only the constancy of W as a figure of merit, then it is shown the "optimum" on-line frequency is 20.6 GHz for conditions representing a cross section of the continental United States. The analysis shows that the frequency choice is site and season dependent. If one desired the best possible accuracy, one might consider tailoring the on-line frequency for local meteorological conditions.

While this work is suggestive, it is by no means complete. A rigorous analysis of the optimum frequency problem would solve for both operating frequencies, f_1 and f_2, of the WVR. It would be done by including (1) a noise model of instrumental performance, (2) the finite bandwidth of each measurement channel, and (3) a realistic weighting of the liquid water retrieval. The last item is perhaps the most difficult with which to deal.

It is clear that the off-line measurement is required solely to deal with the possible presence of liquid water in the form of clouds. In clear sky conditions the off-line channel simply adds noise to the inversion. We have assumed that the spectrum of liquid water varies as frequency squared, a good assumption if the drop size is small. As the drop size grows to be an appreciable fraction of a wavelength, this assumption is violated as the frequency behavior is much more complex. The violation of this assumption is felt most strongly at the highest frequency measurement channel. The analysis must include a realistic estimate of the drop size distribution and occurrence statistics.

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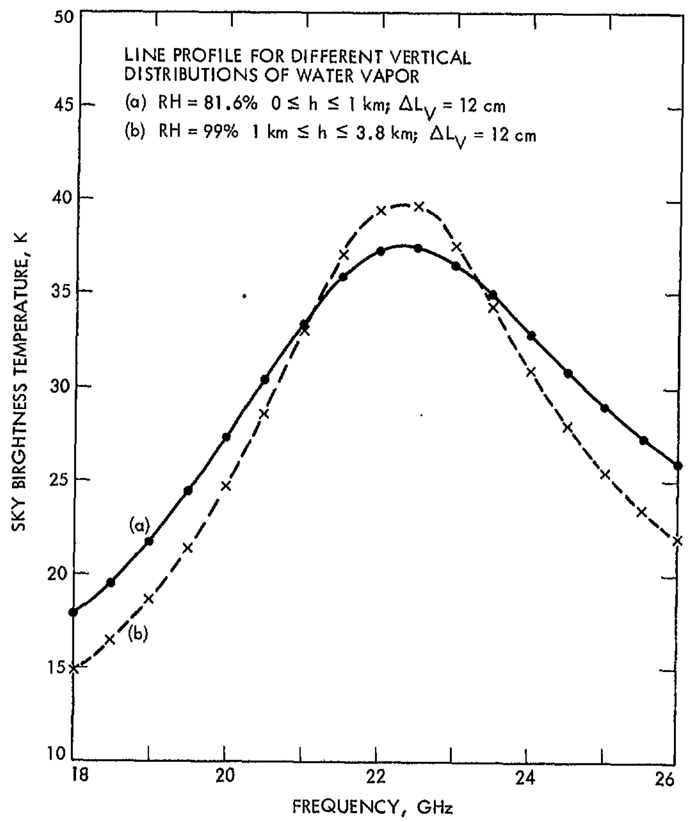

Fig. 1. Line profile of atmospheric water vapor for two different vertical distributions in a standard atmosphere: (a) RH = 81.6% for $0 \leq H \leq 1000$ m; and (b) RH = 99% for $1000 \leq H \leq 3800$ m. In both cases ΔL_v = 12 cm

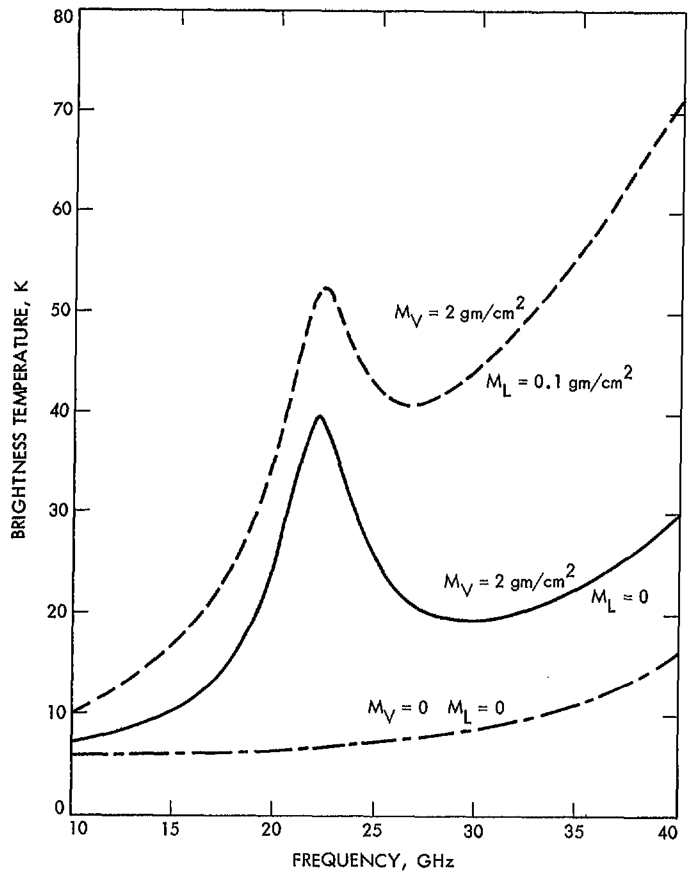

Fig. 2. Brightness temperature of the atmosphere for three cases: (a) oxygen only, no vapor and no liquid; (b) oxygen plus 2 g/cm^2 of precipitable vapor; (c) oxygen plus vapor plus 0.1 g/cm^2 of liquid (as in a cloud)

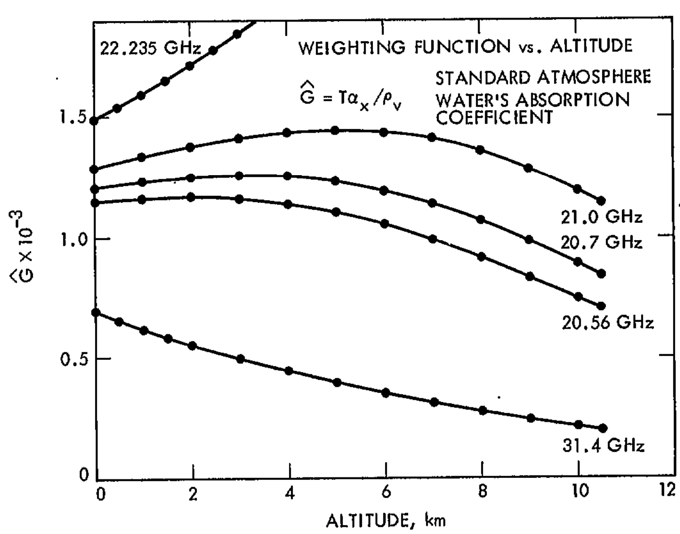

Fig. 3. Variation of the single frequency vapor weighting function versus altitude for several frequencies on or near the water vapor emission line

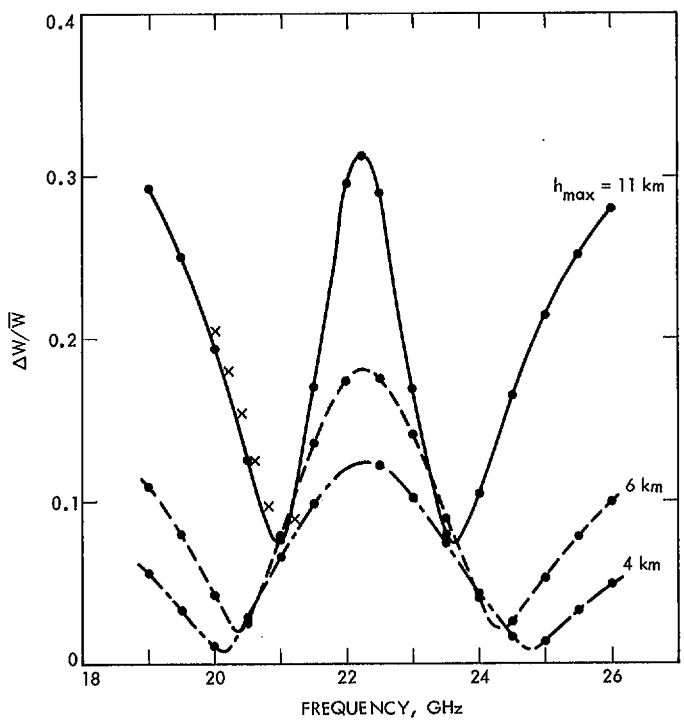

Fig. 4. Variation of the vapor weighting function in a standard atmosphere as a function of frequency for various cutoff altitudes. The off-line frequency is 31.4 GHz.

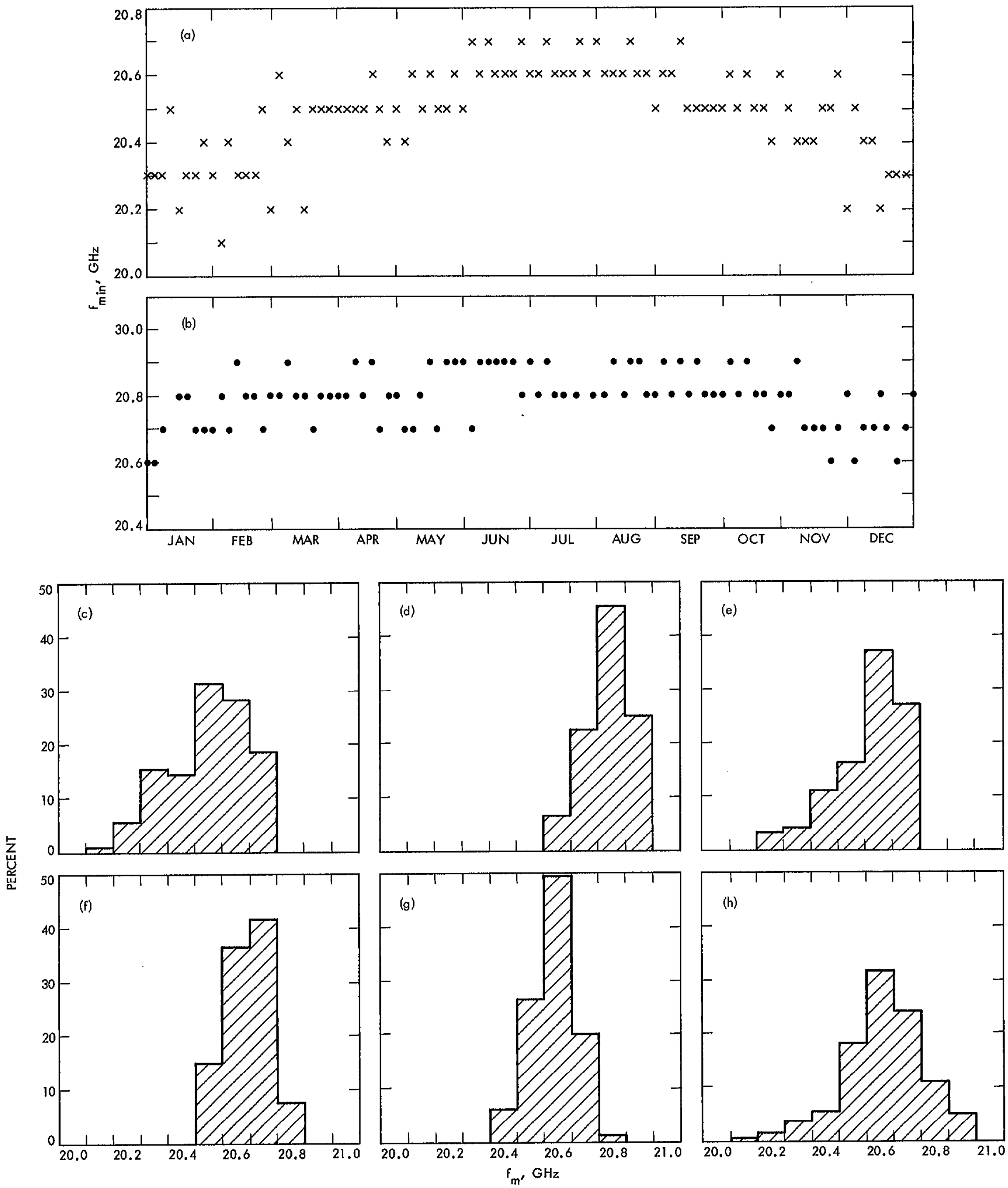

Fig. 5. f_m = **the frequency (GHz) of minimum variation in the weighting function (i.e., minimum $\Delta W/\overline{W}$) versus time for (a) Portland, Maine; (b) El Paso. Also shown are the histograms for (c) Portland; (d) El Paso; (e) Pittsburg; (f) San Diego; (g) Oakland; and (h) all sites.**

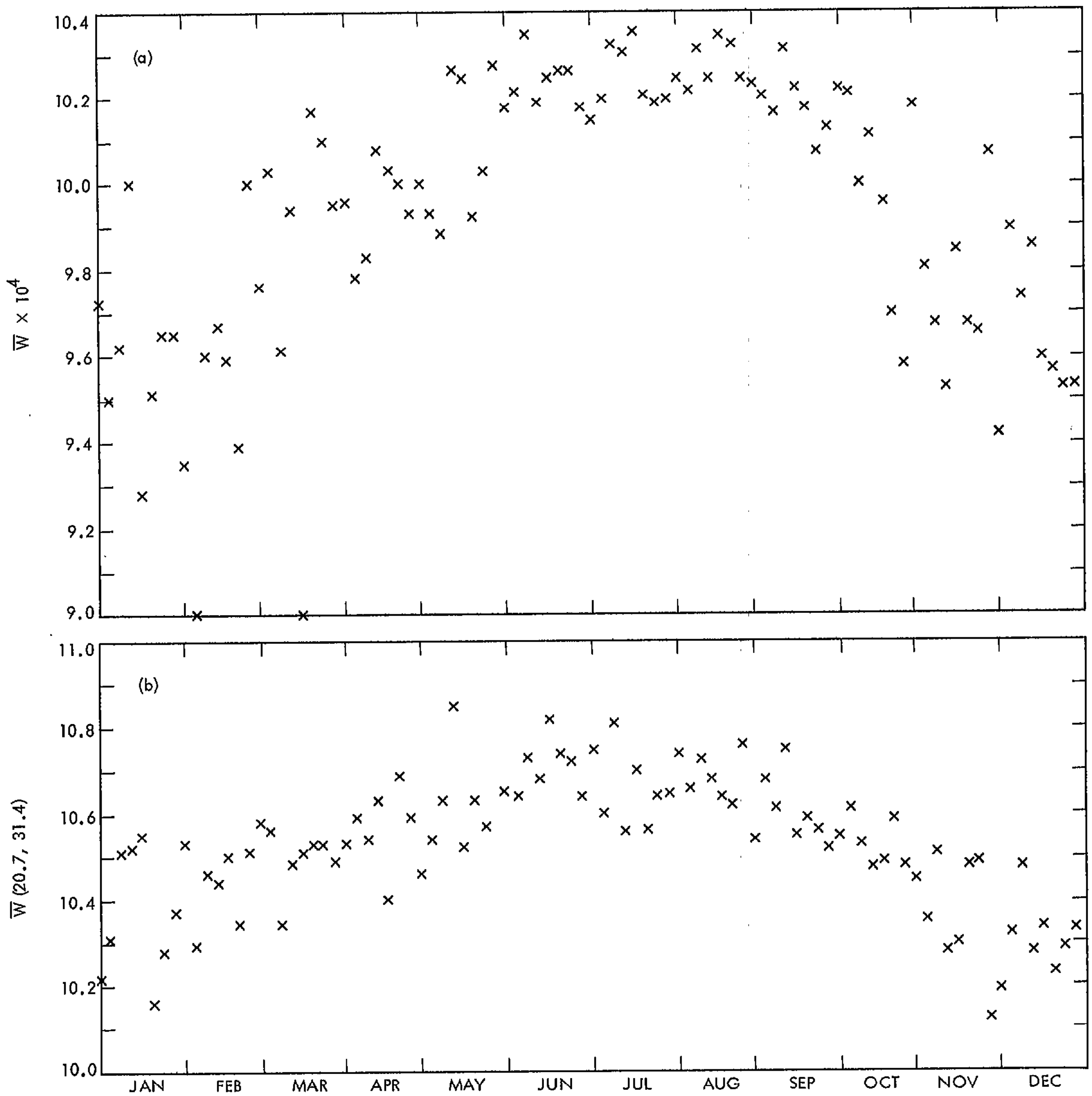

Fig. 6. W = the average value of the weighting function at 20.7 and 31.4 GHz, the frequencies of the JPL water vapor radiometers versus time for (a) Portland; (b) El Paso; and (c) histogram of *W* from all sites

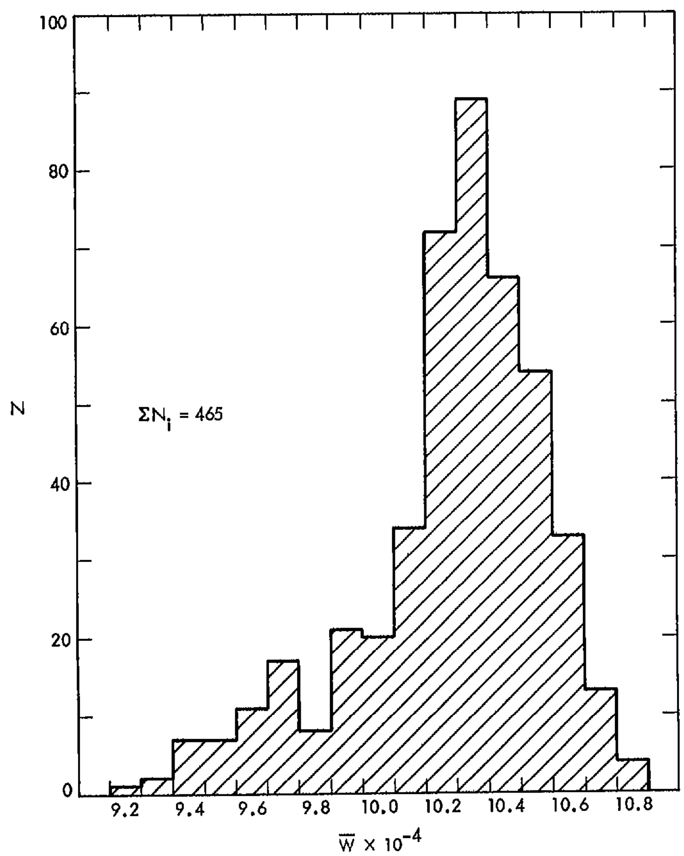

Fig. 6 (contd)